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Interface energy of two-band superconductors

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Using the Ginzburg-Landau theory for two-band superconductors, we determine the surface energy σ_s between coexisting normal and superconducting states at the thermodynamic critical magnetic field. Close to the transition temperature, where the Ginzburg-Landau theory is applicable, we demonstrate that the two-band problem maps onto an effective single band problem. While the order parameters of the two bands may have different amplitudes in the homogeneous bulk, near T_c the Josephson-type coupling between the bands leads to the same spatial dependence of both order parameters near the interface. This finding puts into question the possibility of intermediate, so-called *type-1.5* superconductivity, in the regime where the Ginzburg-Landau theory applies.

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I. INTRODUCTION

Depending on the behavior in external magnetic fields, superconductors are classified as type-I or type-II. In type-I superconductors, the surface energy density σ_s between regions of finite and vanishing order parameters, coexisting at the thermodynamic critical field H_c, is positive.[1] In type-II superconductors this energy is negative and a homogeneous superconducting state is no longer stable, leading to the formation of a vortex lattice.[2] Within the Ginzburg-Landau (GL) theory[1] for one-band superconductors the interface energy per unit area,

$$\sigma_s = \lambda \frac{H_c^2}{4\pi} \Upsilon(\kappa), \tag{1}$$

is determined by the value of the thermodynamic critical field, H_c, the magnetic penetration depth, λ, and the dimensionless function, $\Upsilon(\kappa)$, that depends on the GL parameter $\kappa = \lambda/\xi$, the ratio of the penetration depth and the superconducting coherence length. Properties of $\Upsilon(\kappa)$ are discussed, e.g., in Ref. 3. In the regimes of extreme type-I and type-II superconductivity

$$\Upsilon(\kappa) = \begin{cases} \dfrac{2^{3/2}}{3}\kappa^{-1} & \text{if } \kappa \ll 1 \\ -\dfrac{4}{3}(\sqrt{2}-1) & \text{if } \kappa \gg 1. \end{cases} \tag{2}$$

We have evaluated this function numerically and the result is shown in Fig. 1. The transition between type-I and type-II behaviors occurs for $\kappa = 2^{-1/2}$, where $\Upsilon(\kappa)$ changes sign.

Fermi surfaces in many superconductors may consist of two or more well separated sheets with different energy gaps.[4,5] Evidence for two energy gaps was obtained in high-purity superconducting Nb, Ta, and V,[6] and Nb-doped $SrTiO_3$.[7] Recently, tunneling[8–10] and point contact[11,12] spectroscopies, as well as heat-capacity measurements[13–15] for MgB_2 (Refs. 16–20) give clear evidence for two-band superconductivity with gaps $\Delta_1 \simeq 0.7$ meV and $\Delta_2 \simeq 2.5$ meV (for recent reviews, see Refs. 21 and 22). Other systems that have been discussed as two-band superconductors are RNi_2B_2C with $R = Lu, Y$,[23,24] 2H-$NbSe_2$,[25] and the recently discovered FeAs superconductors.[26] In all cases the amplitude of the superconducting gap is different for different sheets of the Fermi surface.

Motivated by the study of these multiband superconductors, the term *type-1.5* superconductivity has been coined[27] to emphasize the possibility of a state that is intermediate between the two regimes. Specifically, one considers two-component or two-band systems with order parameters $\Psi_1(\mathbf{r})$ and $\Psi_2(\mathbf{r})$ that have qualitatively different spatial dependence, with different respective coherence lengths ξ_1 and ξ_2. The existence of these two length scales emerges from the assumption that one can neglect the Josephson-type coupling between two order parameters. The regime where one expects novel behavior is obviously the limit $\xi_1 \ll \lambda \ll \xi_2$. Then one order-parameter component might behave as a type-I superconductor while the other follows the type-II behavior. Consequences of such behavior were discussed in Ref. 28 where it was concluded that properties emerge that fall outside the usual type-I/type-II dichotomy. For example, the emergence of "vortex molecules" and of an inhomogeneous

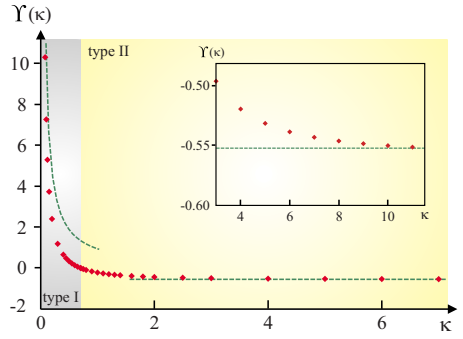

FIG. 1. (Color online) The function, $\Upsilon(\kappa)$, of the one-band problem calculated numerically (full points) compared to theoretical limit of Eq. (2) (dashed lines). The inset shows an enlargement of the large κ domain.

state comprising a mixture of domains of a two-component Meissner state and vortex clusters were proposed. In Ref. 29 the surface energy for such a system was analyzed with the conclusion that $\Upsilon(\kappa)$ must be replaced by a function that depends on several dimensionless quantities, in particular, on the ratio ξ_1/ξ_2. Changing ξ_1/ξ_2 at fixed penetration depth was shown to yield a sign change in σ_s.

Obviously, even in multiband superconductors, the sign of the surface energy is either positive or negative. Thus, it seems more appropriate to discuss the physics that was investigated in Ref. 27 within the GL approach,[30] as interesting modifications of type-II superconductivity. More importantly, it is crucial to analyze what exactly happens in a multiband superconductor in the vicinity of the transition temperature, with

$$\tau = (T_c - T)/T_c \ll 1, \tag{3}$$

in the regime where the GL approach is valid (ignoring, as usual, critical fluctuations).

One of the key features of the two-band GL model is the Josephson-type coupling between the two bands,

$$f_c(\mathbf{r}) = -\eta[\Psi_1^*(\mathbf{r})\Psi_2(\mathbf{r}) + \Psi_2^*(\mathbf{r})\Psi_1(\mathbf{r})], \tag{4}$$

in the expansion of the GL free-energy density. References 28 and 29 analyze the limit $\eta=0$, but assume that both order parameters, while uncoupled, order at the same temperature. The more realistic regime is clearly the one where the common transition temperature is the consequence of a finite order-parameter coupling η.

In this paper we determine the surface energy σ_s of a two-band GL model including the coupling, Eq. (4), between the bands, i.e., we consider $\eta \neq 0$. We find that in the regime $\tau \ll 1$, where the GL theory provides the correct mean-field description, Eq. (1) continues to be the correct expression for the interface energy with same function $\Upsilon(\kappa)$, which implies that the surface energy still changes sign at $\kappa = 2^{-1/2}$. The multicomponent nature of the order parameter enters the GL κ through the values of λ and

$$\xi = (\xi_1^{-2} + \xi_2^{-2})^{-1/2}. \tag{5}$$

A detailed definition of λ and ξ_i in the two-band problem is presented below. We also find that, while the order parameters may have different amplitudes in the homogeneous bulk, $\Psi_{1,0}$ and $\Psi_{2,0}$, close to the transition temperature, i.e., for small τ, they have the same spatial dependence near the interface. In particular,

$$\frac{\Psi_1(z)}{\Psi_2(z)} = \frac{\Psi_{1,0}}{\Psi_{2,0}} + \mathcal{O}(\tau), \tag{6}$$

i.e., the coupling enforces the same spatial dependence for both components. $\Psi_1(\mathbf{r})$ and $\Psi_2(\mathbf{r})$ vary in space on the single length scale ξ of Eq. (5). Close to a superconducting transition it is then sufficient to introduce only one order parameter to characterize the symmetry broken state. An exception is the case where two completely uncoupled order parameters are accidentally degenerate, i.e., both components accidentally have the exact same transition temperatures T_c while they have, at the same time, different coherence lengths. This is the scenario considered in Refs. 28 and 29. We stress that our results do not preclude unconventional type-II behavior that may occur deeper in the ordered state away of the GL domain. This is however beyond the limit of applicability of the GL theory. In the next section we present our analysis. We summarize our findings in Sec. III.

II. TWO-BAND GINZBURG-LANDAU THEORY

We start with the free energy,

$$F = \int f(\mathbf{r}) d^3 r, \tag{7}$$

of a two-band superconductor. F is a functional of the pairing wave functions Ψ_1 and Ψ_2 of the two components or bands and of the vector potential $\mathbf{A}$ associated with the magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$. The free-energy density, $f(\mathbf{r})$, relative to the zero-field normal-state value, is

$$f(\mathbf{r}) = f_1(\mathbf{r}) + f_2(\mathbf{r}) + f_c(\mathbf{r}) + \frac{B^2(\mathbf{r})}{8\pi}. \tag{8}$$

Here the $f_j(\mathbf{r})$, with $j=1,2$, are the GL expansions of the two bands,

$$f_j = a_j|\Psi_j|^2 + \frac{1}{2}b_j|\Psi_j|^4 + \frac{1}{2m_j^*}\left|\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\mathbf{A}\right)\Psi_j\right|^2 \tag{9}$$

and $f_c(\mathbf{r})$ is the coupling term given in Eq. (4). Here $b_j > 0$ and the bands' effective masses are m_j. The physical values of the order parameter and vector potential are determined via $\delta F/\delta\Psi_i = \delta F/\delta A_\alpha = 0$. In principle additional coupling terms such as $(\Psi_1^*\Psi_2)^2$, etc., are allowed. For clean multiband systems, a weak-coupling expansion yields that the coefficients of such terms vanish due to momentum conservation.[33] In addition, even if present, such terms are subleading close to the transition temperature point when compared to $f_c(\mathbf{r})$ of Eq. (4).

We first discuss the homogeneous, zero-field solution. Ignoring the interband coupling, f_c, one finds, as usual, $\Psi_{i,0}(\eta=0) = \sqrt{-a_i/b_i}$ for $a_i < 0$ and $\Psi_{i,0}(\eta=0) = 0$ for $a_i > 0$. In the general case of $f_c \neq 0$, however, the common critical temperature T_c is not equal to either of $T_{c,j}$ and there is no reason that both coefficients $a_i(T)$ change sign at the same temperature. At T_c the smallest eigenvalue of the matrix of the homogeneous quadratic terms in $f(\mathbf{r})$ vanishes. In our problem, this eigenvalue is

$$r_- = \frac{1}{2}(a_1 + a_2 - \sqrt{(a_1 - a_2)^2 + 4\eta^2}), \tag{10}$$

it vanishes for $\eta^2 = a_1(T_c)a_2(T_c)$. Thus, it must hold that $a_{1,2}(T_c) > 0$, as r_- would be negative if one of the two a_i is smaller or equal to zero. Thus, close to T_c, $a_i > 0$ and the interband coupling enhances the transition temperature compared to the largest of the $T_{c,j}$ for the $\eta=0$ limit.

To proceed, we introduce the dimensionless ratio

$$t \equiv \frac{\eta^2 - a_1 a_2}{a_1 a_2} \propto \frac{T_c - T}{T_c} \tag{11}$$

that vanishes at T_c (see also the Appendix). It is convenient to eliminate $\eta^2=(1+t)a_1a_2$ in favor of t. Thus, small t naturally corresponds to finite interband coupling η. For small t we have $r_- \simeq -ta_1a_2/(a_1+a_2)$ and the smallest eigenvalue changes sign at $t=0$.

The free-energy minimization of the homogeneous problem for $\eta \neq 0$ leads to the system

$$a_1\Psi_1 + b_1\Psi_1^3 - \eta\Psi_2 = 0,$$

$$a_2\Psi_2 + b_2\Psi_2^3 - \eta\Psi_1 = 0, \tag{12}$$

which is readily reduced to a fourth-order equation for Ψ_1^2 that can be solved using known formulas for the roots of a quartic equation. One can simplify the problem by recognizing that the GL theory is only valid in the vicinity of the transition temperature, $t \ll 1$. The homogeneous order parameters can easily be determined to leading order in t,

$$\Psi_{1,0}^2 = u_1 t \quad \text{with} \quad u_1 = \frac{a_2^2 a_1}{a_2^2 b_1 + a_1^2 b_2}, \tag{13}$$

$$\Psi_{2,0}^2 = u_2 t \quad \text{with} \quad u_2 = \frac{a_1^2 a_2}{a_2^2 b_1 + a_1^2 b_2}, \tag{14}$$

where the subscript 0 is to denote the zero-field solution. Hence, the temperature dependence of the order parameters is as expected,

$$\Psi_{j,0}^2 \propto t \propto \frac{T_c - T}{T_c}. \tag{15}$$

We stress that within GL theory there is no reason to go to terms of higher orders in t. Of course, away from T_c corrections can be significant, in particular, for small η but those effects require a microscopic approach based on Bogoliubov-de Gennes equations.[31]

Close to T_c, we can also determine the thermodynamic critical field by imposing $f(H_c)=0$,

$$\frac{H_c^2}{4\pi} = \sum_{j=1}^{2} b_j |\psi_j|^4 = \frac{a_1^2 a_2^2 t^2}{a_2^2 b_1 + a_1^2 b_2}. \tag{16}$$

One can *formally* define the one-band penetration depth as $\lambda_i^{-2}=4\pi e^{*2}\Psi_{i,0}^2/(m_i^* c^2)$. Since the additive superfluid density is proportional to λ^{-2}, the actual London penetration depth is

$$\lambda^{-2} = \lambda_1^{-2} + \lambda_2^{-2}. \tag{17}$$

Using Eq. (14), we obtain

$$\lambda^{-2} = \frac{4\pi e^{*2} a_1 a_2}{c^2} \frac{a_2/m_1^* + a_1/m_2^*}{a_2^2 b_1 + a_1^2 b_2} t. \tag{18}$$

It is now straightforward to set up the formalism to determine the interface energy.

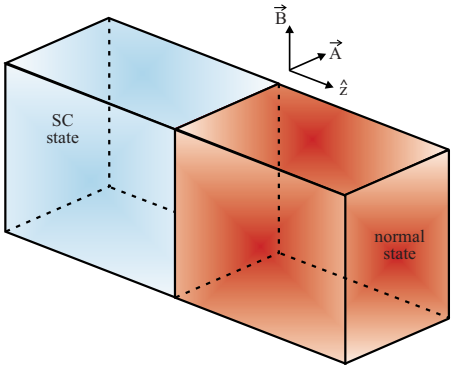

FIG. 2. (Color online) Schematic representation of the interface between the normal and the superconducting states.

III. INTERFACE ENERGY

In evaluating the surface energy we follow closely the classical approach that was used for the single band problem.[1] Consider the interface between superconducting and normal half spaces at the plane $z=0$. The field H is applied along the x axis parallel to the interface and equal to H_c to ensure coexistence of two phases. Then the magnetic induction has only one component $B_x=B(z)$ and the vector potential can be chosen as $A_y=-A(z)$, as shown in Fig. 2, yielding

$$B(z) = A'(z). \tag{19}$$

We can choose real order parameters that vary along the z direction. The Gibbs free energy per unit area that is minimum in a given applied field reads

$$G = F - \frac{H_c}{4\pi}\int_{-\infty}^{\infty} dz B(z). \tag{20}$$

Far away from the interface we have on one side the normal state with $\Psi_i(z\to-\infty)\to 0$ and $B(z\to-\infty)\to H_c$ while on the other side for $z\to\infty$ we have the homogeneous superconducting state with $B\to 0$ and $\Psi_i\to\Psi_{i,0}$.

It is convenient to introduce dimensionless quantities,

$$\psi_j^2 = \frac{\Psi_j^2}{u_j t}, \quad b = \frac{B}{\sqrt{2}H_c}, \quad \text{and} \quad s = \frac{z}{\lambda}, \tag{21}$$

which imply that the dimensionless vector potential $a=A/(\sqrt{2}H_c\lambda)$. The surface energy is then given by

$$\sigma_s = \lambda \frac{H_c^2}{4\pi} \Sigma[\psi_1, \psi_2, a], \tag{22}$$

where Σ is a functional that must be minimized with respect to the ψ_1, ψ_2, and a to yield σ_s. After simple algebra we obtain

$$\Sigma = \int \left[V(\psi_1, \psi_2, a) + \sum_i \kappa_i^{-2} \psi_i'^2 + (a' - 2^{-1/2})^2 \right] ds, \tag{23}$$

where

$$V(\psi_1,\psi_2,a)=\frac{\psi_1^2+\psi_2^2-2\sqrt{1+t}\psi_1\psi_2}{t}+\frac{u}{2}\psi_1^4+\frac{1-u}{2}\psi_2^4$$
$$+\frac{\kappa_2^2\psi_1^2+\kappa_1^2\psi_2^2}{\kappa_1^2+\kappa_2^2}a^2. \quad (24)$$

We use here the following notations: $\psi_i'=d\psi_i/ds$, $b=a'=da/ds$, and $u=u_1$, as given in Eq. (13), and

$$\kappa_i=\frac{\lambda}{\xi_i}, \quad \text{with} \quad \xi_i^2=\frac{\hbar^2}{2a_i m_i^* t}. \quad (25)$$

Minimization of Σ gives a system of coupled differential equations for ψ_i, a,

$$\frac{1}{\kappa_i^2}\psi_i''=\frac{1}{2}\frac{\partial V}{\partial \psi_i}, \quad (26)$$

$$a''=\frac{1}{2}\frac{\partial V}{\partial a}. \quad (27)$$

Multiplying Eq. (26) by ψ_i', Eq. (27) by a' and summing, the first integral of this system is obtained,

$$\sum_i \kappa_i^{-2}\psi_i'^2+a'^2-V(\psi_1,\psi_2,a)=\text{const}. \quad (28)$$

The peculiar term in our analysis is the first one in $V(\psi_1,\psi_2,a)$ of Eq. (24) that seems to be singular as $t\to 0$. Expanding for small t, we have

$$\frac{\psi_1^2+\psi_2^2-2\sqrt{1+t}\psi_1\psi_2}{t}\simeq\frac{(\psi_1-\psi_2)^2}{t}-\psi_1\psi_2. \quad (29)$$

Thus, close to the transition temperature we must have $\psi_1=\psi_2$.

The origin of this behavior is that the coefficients of the quadratic terms in the GL expansion that do not change sign at the transition temperature. This is caused by the Josephson coupling between the two bands that raises the transition temperature of the coupled system above the transition temperatures of the uncoupled systems. This conclusion is confirmed below by a direct numerical minimization of the full functional Σ of Eq. (23).

Introducing $\psi(s)=\psi_1(s)=\psi_2(s)$, which is equivalent to Eq. (6), one obtains the surface energy functional in the form

$$\Sigma=\int ds[V_0(\psi,a)+\kappa^{-2}\psi'^2+(a'-2^{-1/2})^2] \quad (30)$$

with

$$V_0(\psi,a)=-\psi^2+\frac{1}{2}\psi^4+\psi^2a^2 \quad (31)$$

and effective parameter κ given by

$$\kappa^{-2}=\kappa_1^{-2}+\kappa_2^{-2}. \quad (32)$$

This is the same form of the functional as for the standard one-band surface energy problem.[1,3]

It is worth noting that κ_i enter the surface energy only through the combination κ of Eq. (32). In particular, this leads to Eq. (5) for the correlation length of the two band problem with $\kappa=\lambda/\xi$. Thus, the interface problem is identical to the one of a single band system, leading to Eq. (1) with the same function $\Upsilon(\kappa)$.

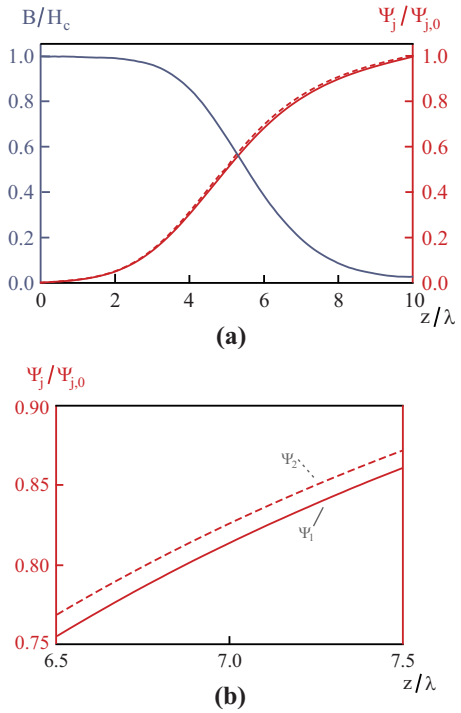

FIG. 3. (Color online) (a) The reduced field and order parameters obtained numerically by minimizing the interface energy functional, Σ, for $\kappa_1=0.45$, $\kappa_2=5$, $u=0.6$, and $t=0.07$. The reduced order parameter ψ_1 is shown by a solid line and the dashed line is ψ_2. (b) The closeup of the order parameters for $6.5<z/\lambda<7.5$.

These conclusions are supported by numerical minimization of $\Sigma[\psi_1,\psi_2,a]$. We discretized the interval $s=[0,2L]$ to N equidistant steps ($s_j=2jL/N$) and minimized Σ with respect to $\psi_1(s_j)$, $\psi_2(s_j)$, and $a(s_j)$ subject to boundary conditions $a'(0)=2^{-1/2}$, $a(2L)=0$, and $\psi_i(0)=0$ and $\psi_i(2L)=\psi_{i,0}$. The homogeneous bulk solutions $\psi_{i,0}$ approach the value $\psi_{i,0}=1$ for $t\to 0$. Finally, since in the limit $2L\to\infty$ the interface position is arbitrary, at $z=L$ we assumed $\psi_1(L)=\frac{1}{2}$, which centers the interface position in the large κ limit.

Our results for $N=400$ are shown in Figs. 3–6. In comparing Figs. 3 and 4, as well as Figs. 5 and 6, we show that the order parameters do indeed approach the behavior with identical spatial variation, as given by Eq. (6), as the critical temperature is approached.

In Figs. 3 and 4 we focus on the most nontrivial limit with $\kappa_1=0.45<2^{-1/2}<\kappa_2=5$. Naively, one could expect ψ_1 to change on distances of the order $\xi_1>\sqrt{2}\lambda$ (type-I behavior) while $\xi_2<\sqrt{2}\lambda$ suggests type-II behavior of ψ_2. Contrary to this expectation we find that both order parameters are strongly coupled by the Josephson energy and have increasingly similar spatial variation as $t\to 0$. As we will see below, the interface energy for this set of parameters is positive and the system behaves as a type-I superconductor as κ in Eq. (32) is dominated by the smallest of the two κ_i.

In Figs. 5 and 6 we show the behavior for $\kappa_1=3$ and $\kappa_2=4$, corresponding indeed to a type-II superconductor with

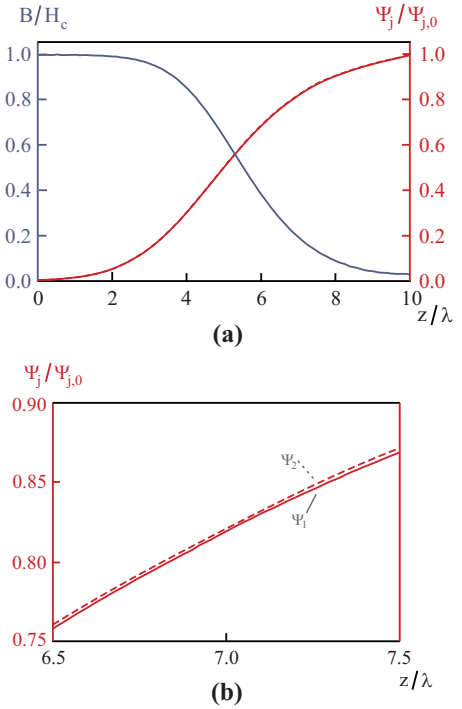

FIG. 4. (Color online) (a) The reduced field and order parameters obtained numerically by minimizing the interface energy functional, Σ, for $\kappa_1=0.45$, $\kappa_2=5$, $u=0.6$, and $t=0.01$. The reduced order parameter ψ_1 is shown by a solid line and the dashed line is ψ_2. (b) The closeup of the order parameters for $6.5<z/\lambda<7.5$.

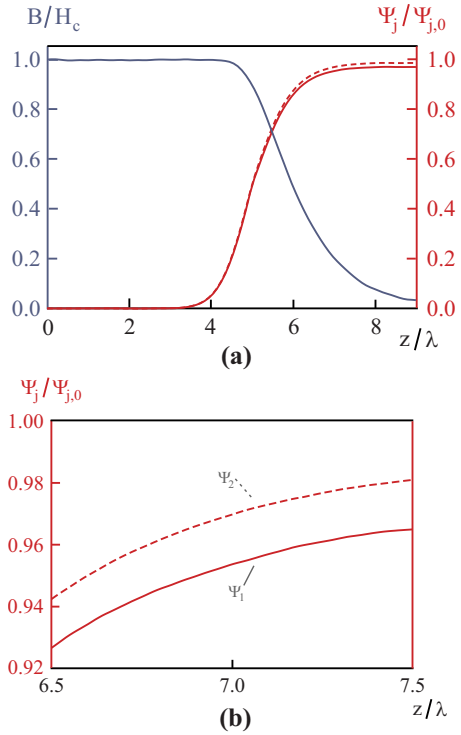

FIG. 5. (Color online) (a) The reduced field and order parameters obtained numerically by minimizing the interface energy functional, Σ, for $\kappa_1=3$, $\kappa_2=4$, $u=0.6$, and $t=0.2$. The reduced order parameter ψ_1 is shown by a solid line and the dashed line is ψ_2. (b) The closeup of the order parameters for $6.5<z/\lambda<7.5$.

negative interface energy (see below for explicit values). Again, both order parameters follow the same spatial dependence and behave according to Eq. (6) as t decreases.

In addition, as t decreases, the value of the minimized functional of Eq. (23) approaches the value of the function $\Upsilon(\kappa)$ of the single band problem with κ determined by Eq. (32). This can explicitly be seen in the numerical results of $\Sigma_{\min}$ and $\Upsilon(\kappa)$ corresponding to Figs. 3–6. The effective one-band solution with $\kappa=0.448$, thus corresponding to Figs. 3 and 4, gives $\Upsilon(\kappa)=0.479$, which differs from the numerical result of Fig. 3, $\Sigma_{\min}=0.530$, by $\sim 11\%$. This difference decreases to $\sim 2\%$ for a smaller value of t as shown in Fig. 4, for which $\Sigma_{\min}=0.488$. The numerical solutions shown in Fig. 5 obtained for $t=0.2$ correspond to $\Sigma_{\min}=-0.275$ whereas the effective one band $\kappa=2.4$ yields $\Upsilon(\kappa)=-0.47$; hence $\Sigma_{\min}$ and Υ differ by $\sim 42\%$. Again, by decreasing the value of t to 0.01, we find results shown in Fig. 6 corresponding to $\Sigma_{\min}=-0.459$, now only by $\sim 2\%$ different from Υ.

IV. SUMMARY

In summary, for a two-band superconductor we analyzed the energy of the interface between regions of a finite order parameter and zero-order parameter, coexisting at the thermodynamic critical field H_c. If one includes the interband Josephson coupling between the bands, i.e., the leading allowed interaction between the Cooper-pair wave function Ψ_1 and Ψ_2, both order parameters vary close to the transition temperature on identical length scales. Thus, despite the fact that both order parameters may have very different amplitudes, they vary on the same characteristic length scale $(\xi_1^{-2}+\xi_2^{-2})^{-1/2}$, where the ξ_i are the typical length scales where the gradient (or kinetic) energies in the GL functional become comparable to the bulk condensation energy. We stress that $\xi_{1,2}$ are just auxiliary quantities and only ξ is a measurable physical length. An important implication of this result is that the surface energy is determined by a single GL parameter $\kappa=\lambda/\xi$ in a way identical to the single band case.

Thus, there is no room left for so-called *type-1.5* superconductivity in the GL regime close to T_c. Of course, our analysis cannot rule out the possibility of interesting unconventional physics due to distinct characteristic length scales ξ_i deep in the superconducting state. Such possibility then requires an approach within the microscopic framework of the Bogoliubov-de Gennes or Gor'kov formalisms. In the recent review by Brandt and Das,[32] situations are described which do not fit to a rigid type-I-type-II dichotomy. Close to T_c, however, only one relevant superconducting order parameter exists and the phenomenology of the transition between type-I and type-II superconductivities is unchanged by the multiband character of the system.

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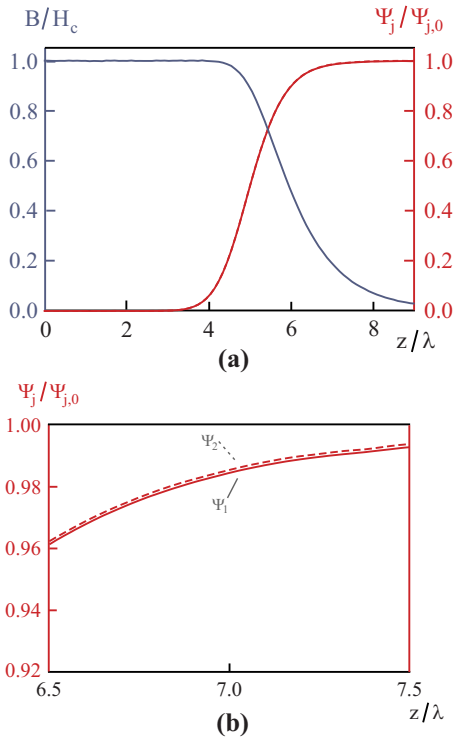

FIG. 6. (Color online) (a) The reduced field and order parameters obtained numerically by minimizing the interface energy functional, Σ, for $\kappa_1=3$, $\kappa_2=4$, $u=0.6$, and $t=0.01$. The reduced order parameter ψ_1 is shown by a solid line and the dashed line is ψ_2. (b) The closeup of the order parameters for $6.5<z/\lambda<7.5$.

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APPENDIX: MICROSCOPIC EXPRESSION OF THE GL COEFFICIENTS

Within a weak-coupling BCS theory it is possible to derive the parameters of the GL expansion, Eqs. (7)–(9), in terms of the microscopic densities of states and pairing interactions,[33]

$$a_i = N_F\left[(\lambda^{-1})_{ii} - n_i\left(\ln\frac{2e^\gamma\omega_D}{\pi T_c} + \tau\right)\right],$$

$$b_i = \frac{7\zeta(3)N_F}{8\pi^2 T_c^2} n_i,$$

$$\eta = \frac{N_F}{\det\hat{\lambda}}\lambda_{12}. \tag{A1}$$

Here N_F is the densities of states at the Fermi level per one spin, $n_i=N_{F,i}/N_F$ are relative densities of states on two bands, $\lambda_{ij}=N_F V_{ij}$ are interaction constants proportional to the symmetric matrix V_{ij} responsible for the Cooper pairing,[34] $\tau=(T_c-T)/T_c$, and $(\lambda^{-1})_{11}=\lambda_{22}/\det\hat{\lambda}$, etc., are elements of the matrix inverse to λ_{ij}.

The transition temperature follows from the condition $[a_1 a_2]_{\tau=0}=\eta^2$ that leads to

$$T_c = \frac{2e^\gamma}{\pi}\omega_D \exp(-1/\tilde{\lambda}) \tag{A2}$$

with effective coupling constant

$$\tilde{\lambda} = 2n_1 n_2 \det\hat{\lambda}[n_1\lambda_{11} + n_2\lambda_{22} - \sqrt{(n_1\lambda_{11} - n_2\lambda_{22})^2 + 4n_1 n_2\lambda_{12}^2}]^{-1}. \tag{A3}$$

It is now straightforward to express the variable t in Eq. (11) in terms of τ close to the transition temperature, which shows

$$t = \frac{\det\hat{\lambda}\sqrt{(n_1\lambda_{11} - n_2\lambda_{22})^2 + 4n_1 n_2\lambda_{12}^2}}{\lambda_{12}^2}\tau. \tag{A4}$$

[1] V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. **20**, 1064 (1950).
[2] A. A. Abrikosov, Sov. Phys. JETP **5**, 1174 (1957).
[3] D. Saint-James, G. Sarma, and E. J. Thomas, *Type-II Superconductivity* (Pergamon, New York, 1969).
[4] H. Suhl, B. T. Matthias, and L. R. Walker, Phys. Rev. Lett. **3**, 552 (1959).
[5] V. A. Moskalenko, Fiz. Met. Metalloved. **8**, 503 (1959) [Phys. Met. Metallogr. **8**, 25 (1959)].
[6] L. Y. L. Shen, N. M. Senozan, and N. E. Phillips, Phys. Rev. Lett. **14**, 1025 (1965).
[7] G. Binnig, A. Baratoff, H. E. Hoenig, and J. G. Bednorz, Phys. Rev. Lett. **45**, 1352 (1980).
[8] F. Giubileo, D. Roditchev, W. Sacks, R. Lamy, D. X. Thanh, J. Klein, S. Miraglia, D. Fruchart, J. Marcus, and P. Monod, Phys. Rev. Lett. **87**, 177008 (2001).
[9] M. Iavarone, G. Karapetrov, A. E. Koshelev, W. K. Kwok, G. W. Crabtree, D. G. Hinks, W. N. Kang, E.-M. Choi, H. J. Kim, H. J. Kim, and S. I. Lee, Phys. Rev. Lett. **89**, 187002 (2002).
[10] M. R. Eskildsen, M. Kugler, S. Tanaka, J. Jun, S. M. Kazakov, J. Karpinski, and Ø. Fischer, Phys. Rev. Lett. **89**, 187003 (2002).
[11] P. Szabó, P. Samuely, J. Kačmarčik, T. Klein, J. Marcus, D. Fruchart, S. Miraglia, C. Marcenat, and A. G. M. Jansen, Phys. Rev. Lett. **87**, 137005 (2001).
[12] H. Schmidt, J. F. Zasadzinski, K. E. Gray, and D. G. Hinks, Phys. Rev. Lett. **88**, 127002 (2002).
[13] Y. Wang, T. Plackowski, and A. Junod, Physica C **355**, 179 (2001).
[14] F. Bouquet, R. A. Fisher, N. E. Phillips, D. G. Hinks, and J. D. Jorgensen, Phys. Rev. Lett. **87**, 047001 (2001).

[15] H. D. Yang, J.-Y. Lin, H. H. Li, F. H. Hsu, C. J. Liu, S.-C. Li, R.-C. Yu, and C.-Q. Jin, Phys. Rev. Lett. **87**, 167003 (2001).
[16] I. N. Askerzade and A. Gencer, Solid State Commun. **123**, 63 (2002).
[17] J. Nagamatsu, N. Nakagawa, T. Muranaka, Y. Zenitani, and J. Akimitsu, Nature (London) **410**, 63 (2001).
[18] S. L. Bud'ko, G. Lapertot, C. Petrovic, C. E. Cunningham, N. Anderson, and P. C. Canfield, Phys. Rev. Lett. **86**, 1877 (2001).
[19] Y. Kong, O. V. Dolgov, O. Jepsen, and O. K. Andersen, Phys. Rev. B **64**, 020501(R) (2001).
[20] J. Kortus, I. I. Mazin, K. D. Belashchenko, V. P. Antropov, and L. L. Boyer, Phys. Rev. Lett. **86**, 4656 (2001).
[21] P. C. Canfield, S. L. Bud'ko, and D. K. Finnemore, Physica C **385**, 1 (2003).
[22] X. X. Xi, Rep. Prog. Phys. **71**, 116501 (2008).
[23] M. Heinecke and K. Winzer, Z. Phys. B: Condens. Matter **98**, 147 (1995).
[24] S. V. Shulga, S.-L. Drechsler, G. Fuchs, K.-H. Muller, K. Winzer, M. Heinecke, and K. Krug, Phys. Rev. Lett. **80**, 1730 (1998).
[25] T. Yokoya, T. Kiss, A. Chainani, S. Shin, M. Nohara, and H. Takagi, Science **294**, 2518 (2001).
[26] H. Ding, P. Richard, K. Nakayama, K. Sugawara, T. Arakane, Y. Sekiba, A. Takayama, S. Souma, T. Sato, T. Takahashi, Z. Wang, X. Dai, Z. Fang, G. F. Chen, J. L. Luo, and N. L. Wang, EPL **83**, 47001 (2008).
[27] V. Moshchalkov, M. Menghini, T. Nishio, Q. H. Chen, A. V. Silhanek, V. H. Dao, L. F. Chibotaru, N. D. Zhigadlo, and J. Karpinski, Phys. Rev. Lett. **102**, 117001 (2009).
[28] E. Babaev and M. Speight, Phys. Rev. B **72**, 180502(R) (2005).
[29] J.-P. Wang, Phys. Lett. A **374**, 58 (2009).
[30] I. N. Askerzade, Phys. Usp. **49**, 1003 (2006).
[31] P.-G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, NY, 1966).
[32] E. Brandt and M. Das, arXiv:1007.1107, Proceedings of International Conference on Superconductivity and Magnetism, 25–30 April, 2010, Antalia, Turkey (to be published).
[33] M. E. Zhitomirsky and V.-H. Dao, Phys. Rev. B **69**, 054508 (2004).
[34] Note that the notation commonly used in the literature $\lambda_{ij}^{(lit)}$ differs from λ_{ij} used here: $\lambda_{ij}^{(lit)}=n_i\lambda_{ij}$. In our definition λ_{ij} continues to be a symmetric (Hermitian) matrix if V_{ij} is symmetric (Hermitian).